AN EXTENSION OF A CARD TRICK

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In my article, "Interest in Mathematics – It's in the Cards," (*Mathematics Teacher*, February 1989) I described some discoveries made by middle school students and Algebra I students about a certain card trick. In this "addendum" to that article, I shall derive some variations on the card trick.

First, here is how the trick goes, as given in the original article: The trickster states that he can prevent three chosen cards from being dealt to the player, even though he does not know what they are. The player begins by drawing any three cards from an ordinary deck without showing them. The trickster then separates the remaining 49 cards face-down in piles of 5, 15, 15, with the other 14 put aside for the moment. The player is told to note the number and suit of each of the three cards he chose, and then to place the first of them (call it F) on top of the pile of 5, "burying" it with several cards from the first pile of 15; place the second of the three chosen cards (call it S) on top of what remains of the first pile of 15, "burying" it with several cards from the second pile of 15; finally place the third chosen card (call it T) on top of what remains of the second pile of 15, and "bury" it with the pile of 14 cards that were put aside. All the cards are face down.

The trickster picks up the piles of cards, still face down, first picking up the pile containing T buried with the 14 cards, placing this pile on top of the pile that contains S, and finally placing all these on top of the pile that contains F. The trickster then proceeds to deal the cards from the top of the deck, giving first to himself, then to the player, and alternating until all the cards are dealt. Each person will have 26 cards. The trickster is careful to keep the cards dealt to himself in the same order in which they were dealt. The trickster deals again, from the cards dealt to himself in the previous deal, again dealing to himself first. Each person now has 13 cards. Again he deals, using only the cards dealt to himself on the previous deal, only this time, he deals to the player first. The trickster will have 6 cards and the player 7. Finally the trickster deals a fourth time, to himself first. Each now has three cards and it will be found that the dealer has retained the three cards originally drawn by the player.

Note that in the original trick, the dealing directions require that the deal is to TRICKSTER FIRST on deal one, TRICKSTER FIRST on deal two, PLAYER FIRST on deal three, and TRICKSTER FIRST on deal four. We will call this TTPT. This addendum will show how the dealing directions can be changed to other combinations, such as PPPP, by changing the numbers of cards in the original piles. (The dealing directions TTPT correspond to original piles 5, 15, 15, 14 aside; it will turn out that the dealing directions PPPP correspond to original piles of 14, 15, 15, 5 aside.)

Let n be the bottom-relative position (brp) of a card at the beginning of the first deal. If n is odd, then $53-n$, the top-relative position (trp) of this card is even, and we can see that dealing to the player first will insure that the dealer retains this card. After deal one, the brp of this card will be $(53-n)/2$, and the trp will be $27-(53-n)/2$. Now, $27-(53-n)/2$ may be even, or it may be odd. Suppose first that it is even. Then the dealing directions would be to player first on deal two, making $(27-(53-n)/2)/2$ the brp of the card after deal two, and $14-(27-(53-n)/2)/2$, which simplifies to $14-27/2+53/4-n/4$, the trp. Again, this last expression may be even, or it may be odd. In the case that it is even, then in deal three, the player gets the first card. After deal three, the brp is $(14-27/2+53/4-n/4)/2$, which simplifies to $7-27/4+53/8-n/8$. On this deal, the dealer received six cards, so the trp is $7-7+27/4-53/8+n/8 = 27/4-53/8+n/8$, which is of course, either even or odd. Taking the case where it is even requires that on deal four, the deal is to the player first. The brp of the card we are following is $27/8-53/16+n/16$, and the dealer now has only three cards. Thus, this last expression is equal to 1, 2, or 3, and we solve

$$\begin{aligned} 27/8 - 53/16 + n/16 &= 1 \\ 54 - 53 + n &= 16 \\ n &= 15. \end{aligned}$$

Similarly, solving $27/8 - 53/16 + n/16 = 2$ gives $n = 31$, and solving $27/8 - 53/16 + n/16 = 3$ gives $n = 47$.

Remember that n stands for the original bottom relative position of a card prior to deal one. In order for n to be 15, 31, and 47, the three chosen cards will have to be placed atop piles of 14, 15, and 15 cards, with 5 aside. We summarize, saying that "variation" PPPP corresponds to 14, 15, 15, 5.

Other "variations" can be derived by taking other cases as to the even-ness or odd-ness of a card's position in the deck after each deal. In deal one, had we considered that $27 - (53-n)/2$ was odd, we would deal to the trickster first on deal two. After deal two, the brp of the card would be $(27 - (53-n)/2+1)/2 = 14-53/4-n/4$, and the trp, $14 - (14-53/4+n/4)$, or $53/4-n/4$. Supposing this latter is even, deal three would be to the player first, and the brp would be $53/8-n/8$. The trp would be $7-53/8-n/8$. If this is even, deal four is to the player first, an the brp of a card in the dealer's hand is $7/2-53/16-n/16$. Setting this equal to 1, 2, and 3, gives $n = 13$, 29, and 45. Thus, the "variation" PTPP corresponds to 12, 15, 15, 7.

In finding each expression to set equal to 1, 2, or 3, we let the original brp of a card be represented by n, then $53-n$ will be the trp prior to deal one. Each time we assume that the trp is even, we deal to the player first; if we assume it is odd, the deal is to the trickster first. The brp of a card on any deal will be found by taking half of its trp after the previous deal, always "rounding up" when the trp from the previous deal was odd. A trp is found by subtracting the brp from 27 on deal two, 14 on deal three, and 7 on deal four. The rationale for these "rules" is discussed in the article "Interest in Mathematics – It's in the Cards."

In all there are twelve variations that work. These twelve correspond to the sixteen combinations of two things (T and P) taken four at a time (for the four deals), with the combinations TTTT, TPTT, PTTT, and PPTT deleted. The deletions are necessary, because these combinations would lead to the trickster's having four cards at the end; deal three gives seven cards to the person dealt first, and if the trickster has these seven and deals first to himself again on the fourth deal, he ends up with four cards instead of the exact three chosen by the player at the beginning.

Table 1 summarizes all the variations. The first six variations came from assuming that the brp of the three chosen cards prior to deal one is even, as was the case in the original article and the variations discussed in detail in this addendum; the last six from assuming it was odd.

TABLE 1.

5, 15, 15, 14 aside*	TTPT
7, 15, 15, 12 aside	TPPT
9, 15, 15, 10 aside	TTTP
11, 15, 15, 8 aside	TPTP
13, 15, 15, 6 aside	TTPP
15, 15, 15, 4 aside	TPPP

4, 15, 15, 15	aside	PTPT
6, 15, 15, 13	aside	PPPT
8, 15, 15, 11	aside	PTTP
10, 15, 15, 9	aside*	PPTP
12, 15, 15, 7	aside	PTPP
14, 15, 15, 5	aside	PPPP

*It is interesting to note that when two different variations have the sum of the numbers of cards in their first piles equal to 15, the dealing directions for the two variations are "opposites" in the sense that P's and T's are interchanged. I would like to see if there is some group theory treatment of this fact.

Reference

Mulligan, Catherine Herr. "Interest in Mathematics–It's in the Cards." *Mathematics Teacher*, 82(February 1989): 100-103.

MATH SCRAMBLER

Unscramble these four mixed-up math terms, one letter to each blank:

S I M U N ___ ___ ___ ☐ ___

H E L O W ___ ___ ___ ☐ ___

T H I G E H ☐ ___ ___ ___ ___ ___

S O N D E C ___ ___ ☐ ___ ☐ ___

Now, rearrange the letters in the boxes to form the answer to the riddle below:

IF I HAVE 2 SANDWICHES AND YOU HAVE 3 SANDWICHES, WHAT DO WE HAVE?

___ ___ ___ ___ ___ !

Answer is on page 17.